



Christ Church
Grammar School

2016
UNIT TEST 4

MATHEMATICS SPECIALIST Year 12

Section One:
Calculator-free

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.
2. Answer all questions.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that **you do not use pencil**, except in diagrams.

Question 1

(6 marks)

$$\frac{3x^2 + 2x + 5}{(x + 1)(x^2 + 2)} = \frac{A}{(x + 1)} + \frac{Bx + C}{(x^2 + 2)}, \text{ where } A, B \text{ and } C \text{ are real numbers.}$$

(a) Determine the values of A , B and C .

(4 marks)

(b) Hence, given that $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$, $a \neq 0$,

determine $\int \frac{3x^2 + 2x + 5}{(x + 1)(x^2 + 2)} dx$

(2 marks)

Question 2

(5 marks)

A particle with position vector $\mathbf{r}(t)$ (in metres) moves such that its velocity (in metres per second) at time t seconds, $t \geq 0$, is given by

$$\mathbf{v}(t) = -4\sin(2t) \mathbf{i} + 6 \cos(2t) \mathbf{j}$$

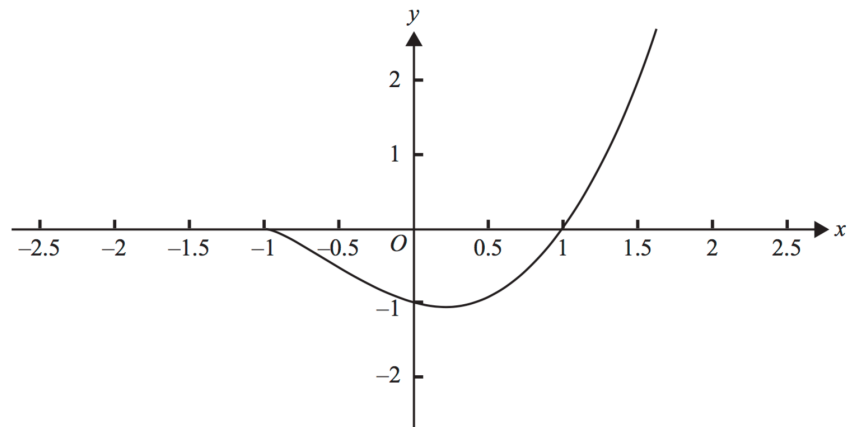
- (a) Given that $\mathbf{r}(0) = 2\mathbf{i}$, determine an expression for the position of the particle at time t . (3 marks)

- (b) Determine the cartesian equation of the path followed by the particle. (2 marks)

Question 3

(4 marks)

Part of the graph with equation $y = (x^2 - 1)\sqrt{x + 1}$ is shown below.



Determine the area of the region bounded by the curve and the x -axis.

Give your answer in the form $\frac{a\sqrt{b}}{c}$, where a , b and c are integers.



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UNIT TEST 4

MATHEMATICS SPECIALIST Year 12

Section Two:

Calculator-assumed

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

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Instructions to candidates

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Question 4

(5 marks)

Let S be the curve in the cartesian plane defined by $\mathbf{r}(t) = (2 - t) \mathbf{i} + (3 + t^2) \mathbf{j}$.

Let T be the curve in the cartesian plane defined by $y = 6x - 10$.

Determine the coordinates of the points in $S \cap T$, the intersection of S and T .

Question 5

(8 marks)

Consider the function $f: [0, \infty) \rightarrow \mathbb{R}$, where $f(x) = \frac{6x\sqrt{x}}{3x^2 + 1}$

The graph of f is rotated about the x -axis between $x = 0$ and $x = \frac{1}{\sqrt{3}}$ to form a solid of revolution with volume V .

(a) Show that $V = 2\pi \int_0^{\frac{1}{\sqrt{3}}} \frac{18x^3}{(3x^2 + 1)^2} dx$ (1 mark)

(b) Use the substitution $u = 3x^2 + 1$ to express V in the form $2\pi \int_a^b \left(\frac{c}{u} + \frac{d}{u^2}\right) du$ (4 marks)

- (c) **Hence**, by using an appropriate antiderivative, determine the exact value of V .
(3 marks)

Question 6**(11 marks)**

The position (in metres) of a projectile at time t seconds, $t \geq 0$, is given by

$$\mathbf{r}(t) = 400t \mathbf{i} + (500t - 5t^2) \mathbf{j}$$

The projectile is fired from a point on the ground.

(a) Find the time taken for the projectile to reach the ground again. **(2 marks)**

(b) Determine the speed at which the projectile hits the ground. **(3 marks)**

(c) Determine the maximum height of the projectile. **(2 marks)**

The distance travelled by the projectile between times $t = a$ and $t = b$ is given by

$$\int_a^b |\mathbf{v}(t)| dt$$

where $|\mathbf{v}(t)|$ is the speed of the projectile at time t .

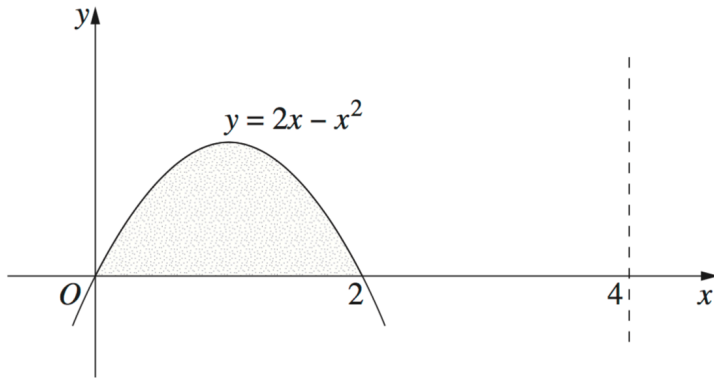
- (d) Find the time taken from when the projectile is fired from a point on the ground to it has completed 80% of the total distance travelled by the projectile. (4 marks)

Question 7

(6 marks)

The shaded region in the diagram below is bounded by the x -axis and the curve $y = 2x - x^2$.

The shaded region is rotated about the line $x = 4$ to form a solid.



Determine the volume of the solid.